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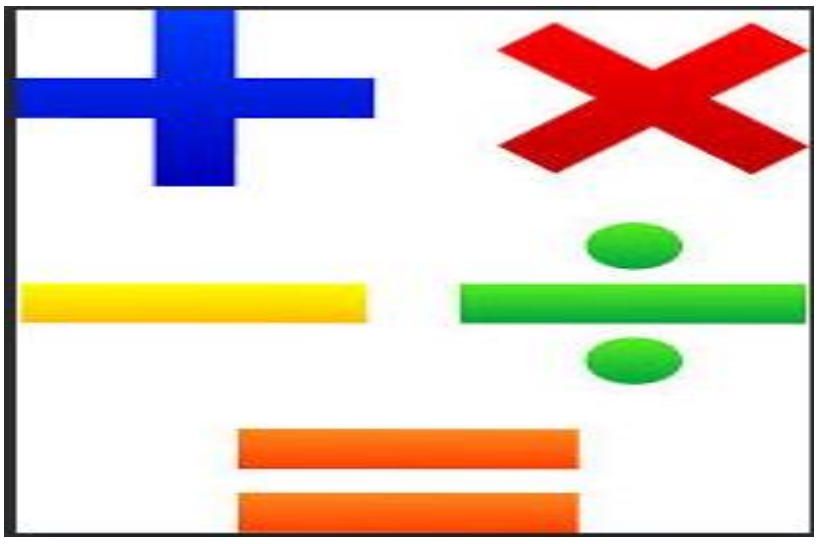
'Trying our best to be our best'

Service and Stewardship. Justice. Peace and Forgiveness
Generosity. Thankfulness. Equality
Love and Compassion

Welbourn Church of England Primary School

Mathematics Calculation Policy



November 2018

Introduction

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

National Curriculum

The National Curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of maths through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- **reason** mathematically by following a line or enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

This policy is a guide for all staff at Welbourn Church of England Primary School and it is expected that teachers will use their professional judgement as to when consolidation of existing skills is required or if to move onto the next stage or concept. However, the focus must always remain on breadth and depth rather than accelerating through concepts. Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by tackling challenging and varied problems.

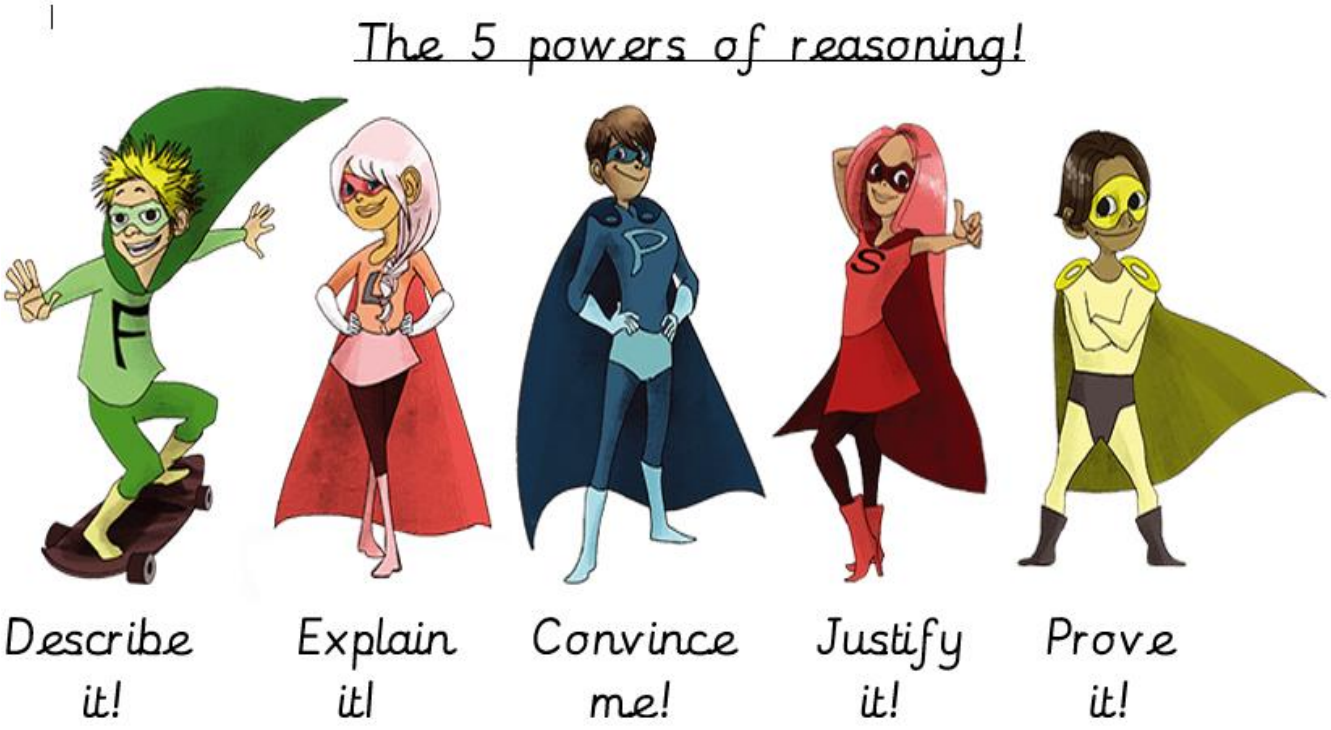
Fluency and conceptual understanding

Teachers are expected to use a variety of resources to enable all pupils to understand mathematical concepts. To support teaching and learning of these calculations, the principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] enables children to have a true understanding of a mathematical concept, they need to master all three phases to fully understands the concept.

As children develop an underlying understanding of key concepts and processes within maths, they will be fully supported in verbalising and explaining their understanding. As children develop a secure understanding of number facts, mental and written methods will become strengthened and refined.

Developing reasoning

Whilst learning key concepts for the 4 operations, children will use reasoning to develop their understanding. This can be developed through these character and skills.



Mathematical Vocabulary

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning and developing reasoning. It is essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct.

(See Mathematical vocabulary booklet for more information)

Stage 1 Addition and Subtraction

Addition stage 1

Points to note: Use the language calculation not ‘sum’ (sum means total or plus)
Use the language digit not ‘number (number is the amount or quantity)

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

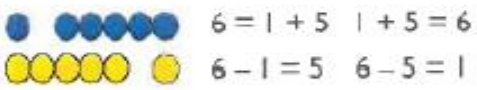
add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will use practical equipment to combine groups of objects to find a total. Practical resources will support children's development of mental pictures and images.

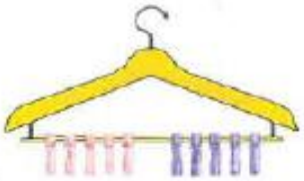
Children will begin to understand commutativity and the principle of exchange. They will be confident in using the terms 'worth' and 'value' when talking about single-digit numbers.

Children can represent calculations using objects and talk about their representations.

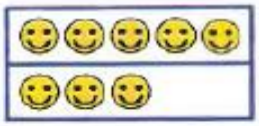
Fingers



Beads or any object



Pegs



Counters

Cubes

Cuisenaire Rods

Numicon

Straws

Subtraction stage 1

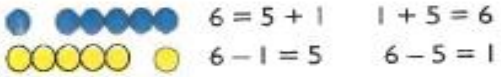
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is../than…?, how much more/less is…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will use practical equipment to physically remove an amount from the group to find the total remaining. Practical resources will support children's development of mental pictures and images.

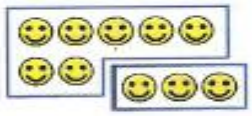
Children can represent calculations using objects and talk about their representations.

Fingers



Beads or any object

Pegs



Counters

Cubes

Cuisenaire Rods

Numicon

Straws

Children will also be introduced to the language of comparison including equal use of the vocabulary 'less' and 'more'.

There are more blue than red.
There are less red than blue.

Cubes

Stage 2 Addition and Subtraction

Addition stage 2

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

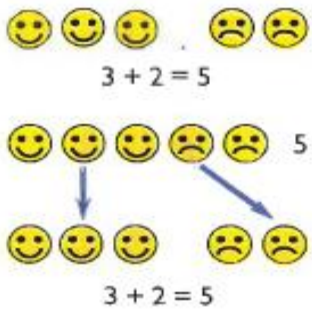
add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

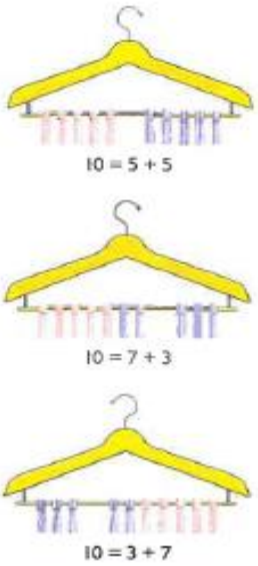
Practical resources will continue to support children's development of mental pictures and images. As these become firm, children will begin to develop ways to represent their mental images and their practical resources using pictures.

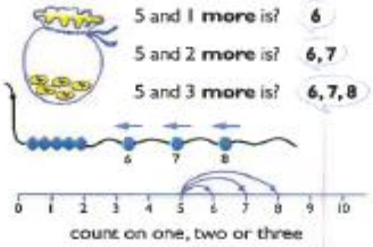
The children will begin to use number sentences alongside their pictures and practical resources.

They will also begin to think and talk flexibly about addition.

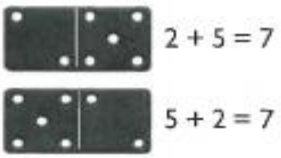
The direct link between addition and subtraction should be made explicit when using models and representations.

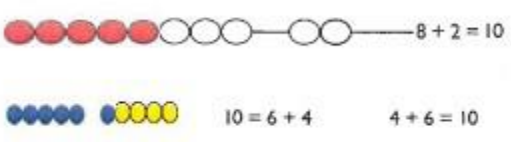


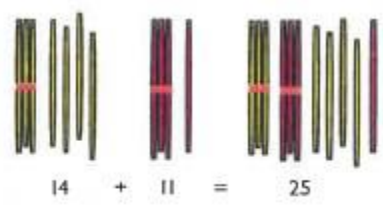


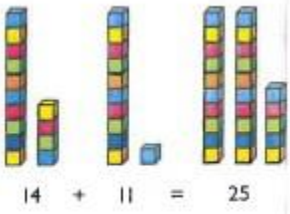


9 and 1 more is 10
9 add 1 equals 10
9 + 1 = 10









Subtraction stage 2

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is.../than...?, how much more/less is...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Practical resources will continue to support children's development of mental pictures and images. As these become firm, children will begin to develop ways to represent their mental images and their practical resources using pictures.

The children will begin to use number sentences alongside their pictures and practical resources.

They will also begin to think and talk flexibly about subtraction and make links to the inverse of addition.

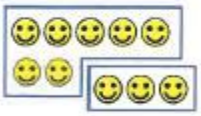
Children will understand that subtraction is not commutative and so the numbers in a calculation can be in any order but will result in a different answer.

The direct link between addition and subtraction should be made explicit when using models and representations.

6 + ? = 10
10 – 6 = ?

? + 6 = 10
10 – 4 = 6

10 – 7 = 3
10 – 3 = 7



10 – 7 = 3
10 – 3 = 7

6 – 2 = 4

Children will continue to be introduced to the language of comparison and its link to finding the difference structure of subtraction.

There are more blue than red.
There are less red than blue.
There are 9 more blue than red.
There are 9 less red than blue.

Stage 3 Addition and Subtraction

Addition stage 3

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

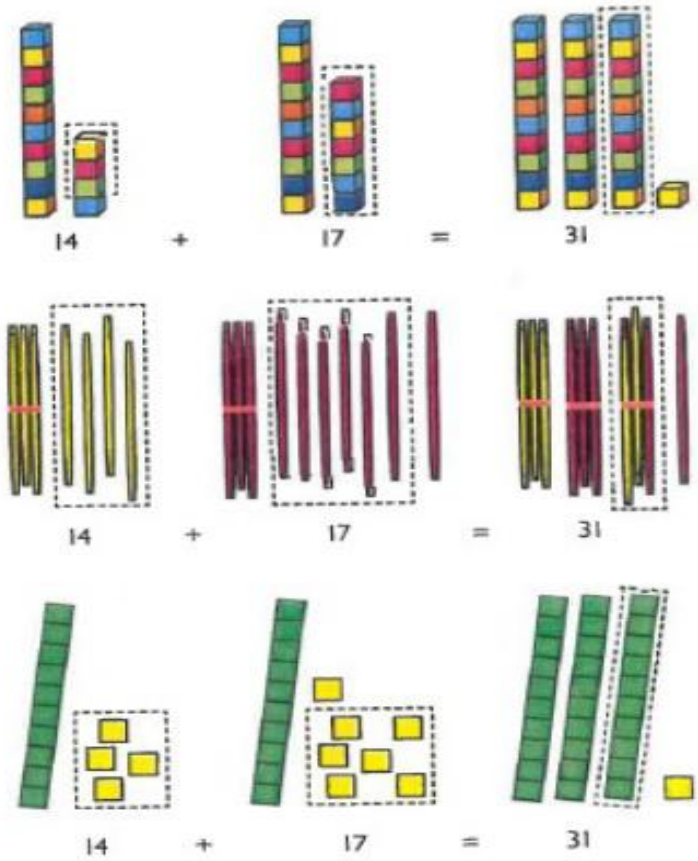
add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will now be confident in using concrete equipment to help them combine groups of objects with numbers up to 20.

They will continue using practical equipment as they begin to also use number tracks, number lines and hundred squares to support their mental methods.

Children will start to work with totals greater than 20 which require them to apply their knowledge of the principle of exchange. They will talk confidently about this.

14 + 17



Subtraction stage 3

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

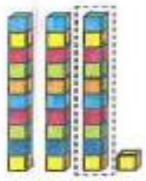
subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is../than…?, how much more/less is…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

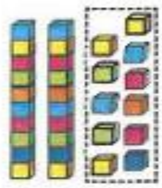
Children will now be confident in using concrete equipment to help them 'take away' and 'find the difference'.

They will continue using practical equipment as they begin to also use number tracks, number lines and hundred squares to support their mental methods.

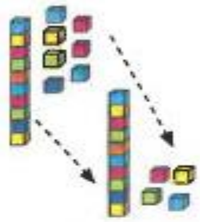
Children will start to work with numbers greater than 20 which require them to apply their knowledge of the principle of exchange. They will talk confidently about this.

31 – 14

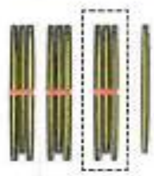


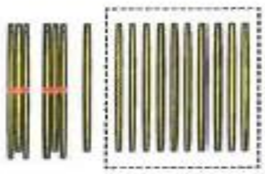


31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the four ones associated with 14

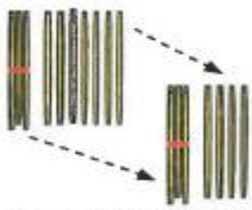


14 can now be removed from the 31 leaving 17

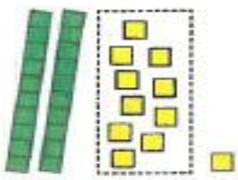




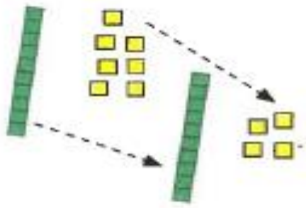
31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the four ones associated with 14



14 can now be removed from the 31 leaving 17



31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the four ones associated with 14



14 can now be removed from the 31 leaving 17

As children become accustomed to repartitioning numbers, they can be introduced to formal notation of the repartitioning.

Points to remember

By the end of Year 2, to enable pupils to meet EXS or Age related Expectation, they need to be able to partition numbers in a variety of ways. Children should not be taught to 'borrow' from the other column. They should be taught to 'exchange' and repartition the numbers in the calculation.

Stage 4 Addition and Subtraction

Addition stage 4

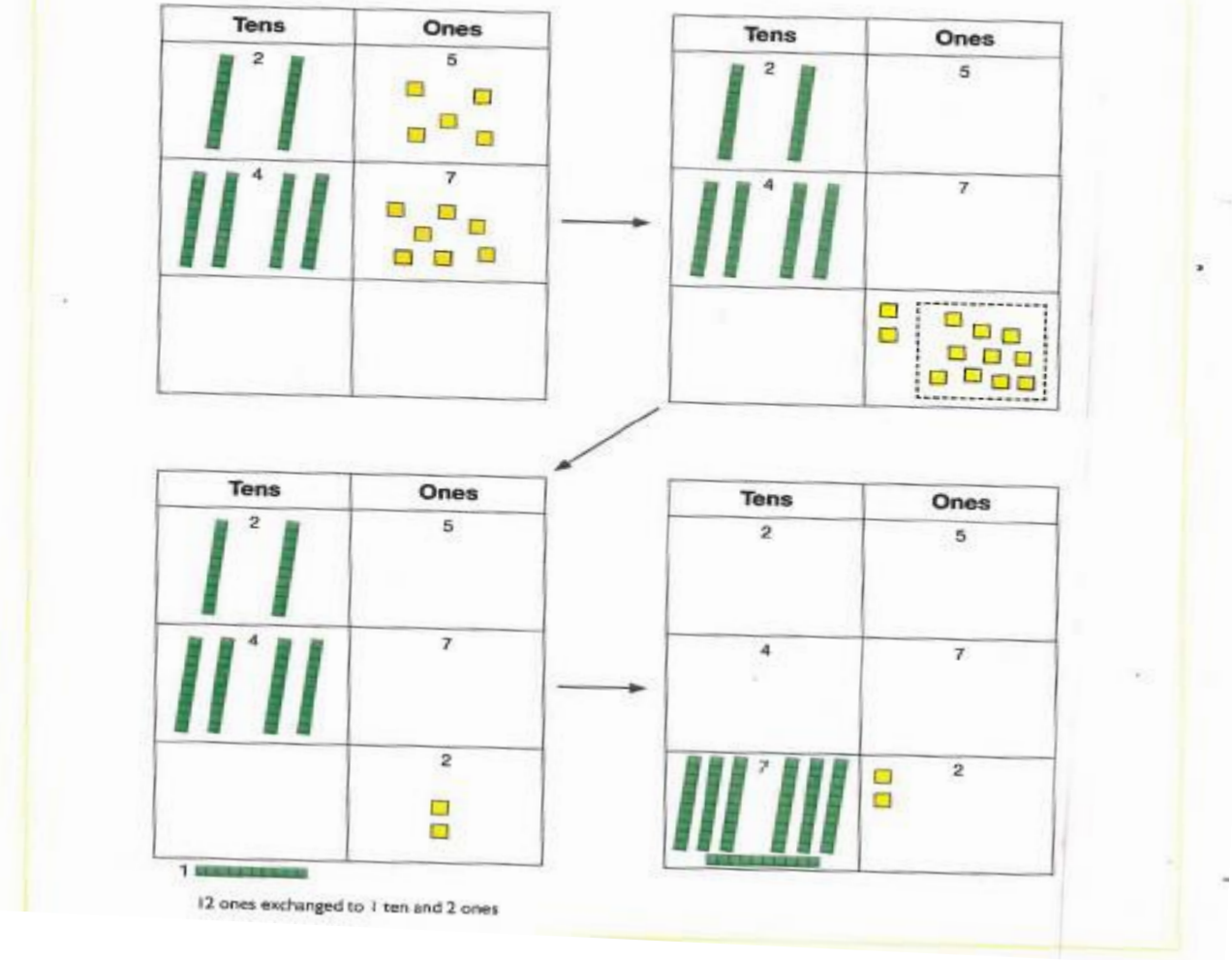
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children are now confident in using concrete equipment to combine objects using the principle of exchange appropriately.

They will now begin to organise their concrete equipment (e.g. Straws, Dienes, Place Value Counters) in a vertical manner where their combined totals are situated at the bottom.

25 + 47



12 ones exchanged to 1 ten and 2 ones

Subtraction stage 4

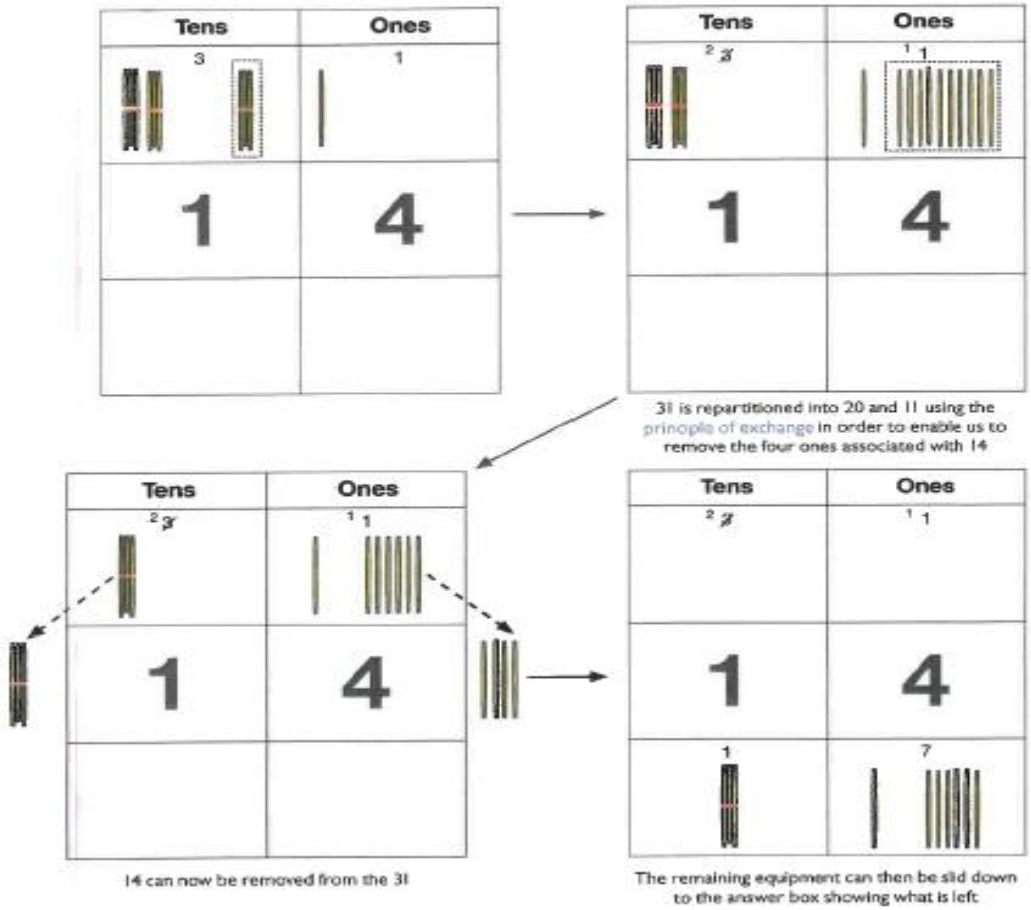
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is../than…?, how much more/less is…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children are now confident in using concrete equipment to 'take away' and 'find the difference' using the principle of exchange appropriately.

They will now begin to organise their concrete equipment (e.g. Straws, Dienes, Place Value Counters) in a vertical manner where the amount that remains at the end of the calculation is situated at the bottom.

31 – 14



Stage 5 Addition and Subtraction

Addition stage 5

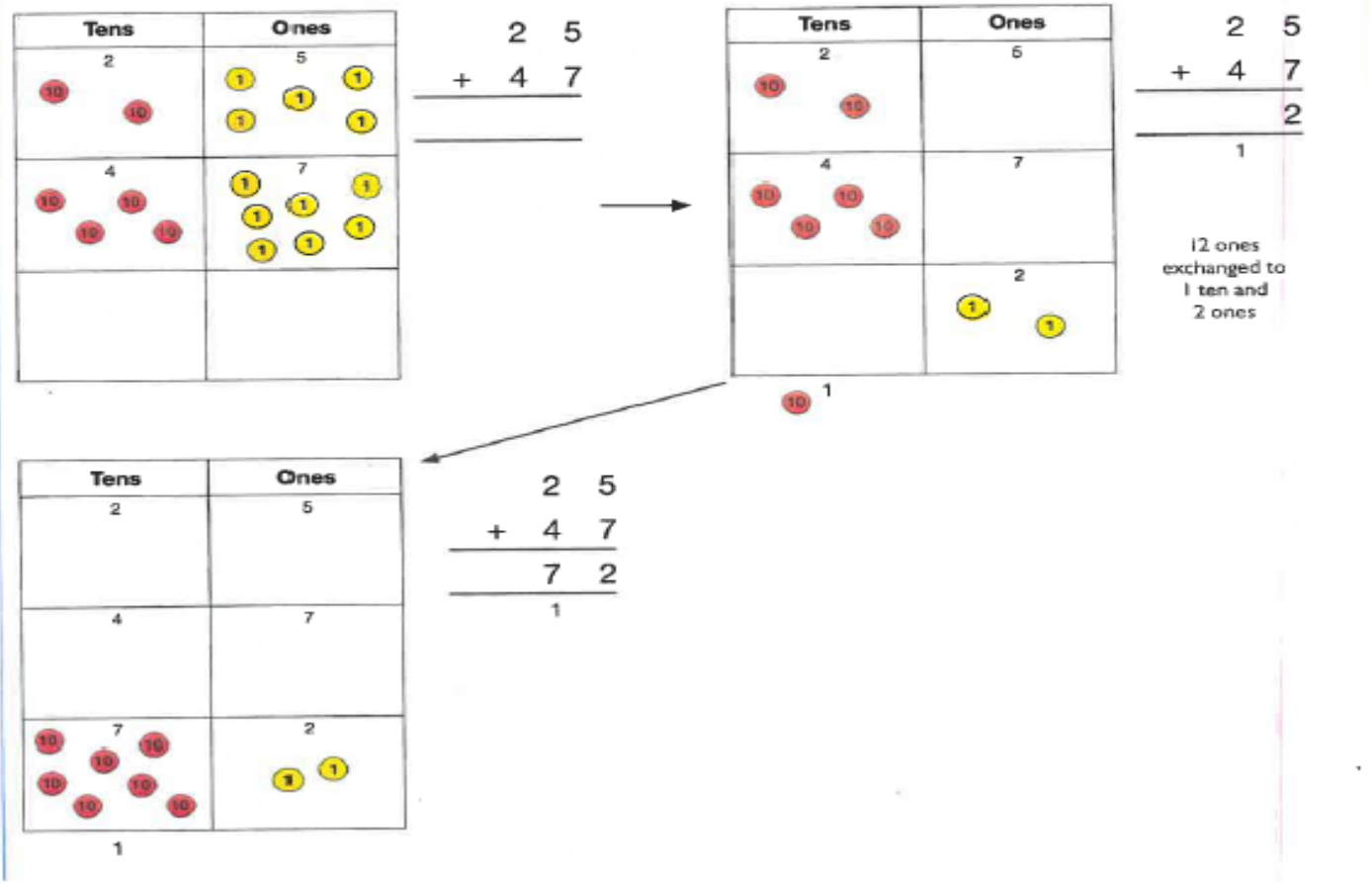
VOCABULARY **Ensure the correct vocabulary is used at all stages of learning**

add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will now be secure in organising their concrete equipment in a vertical manner where their combined totals are situated at the bottom.

They will be now able to make the links between this representation and the formal column addition when seen alongside each other.

25 + 47



Subtraction stage 5

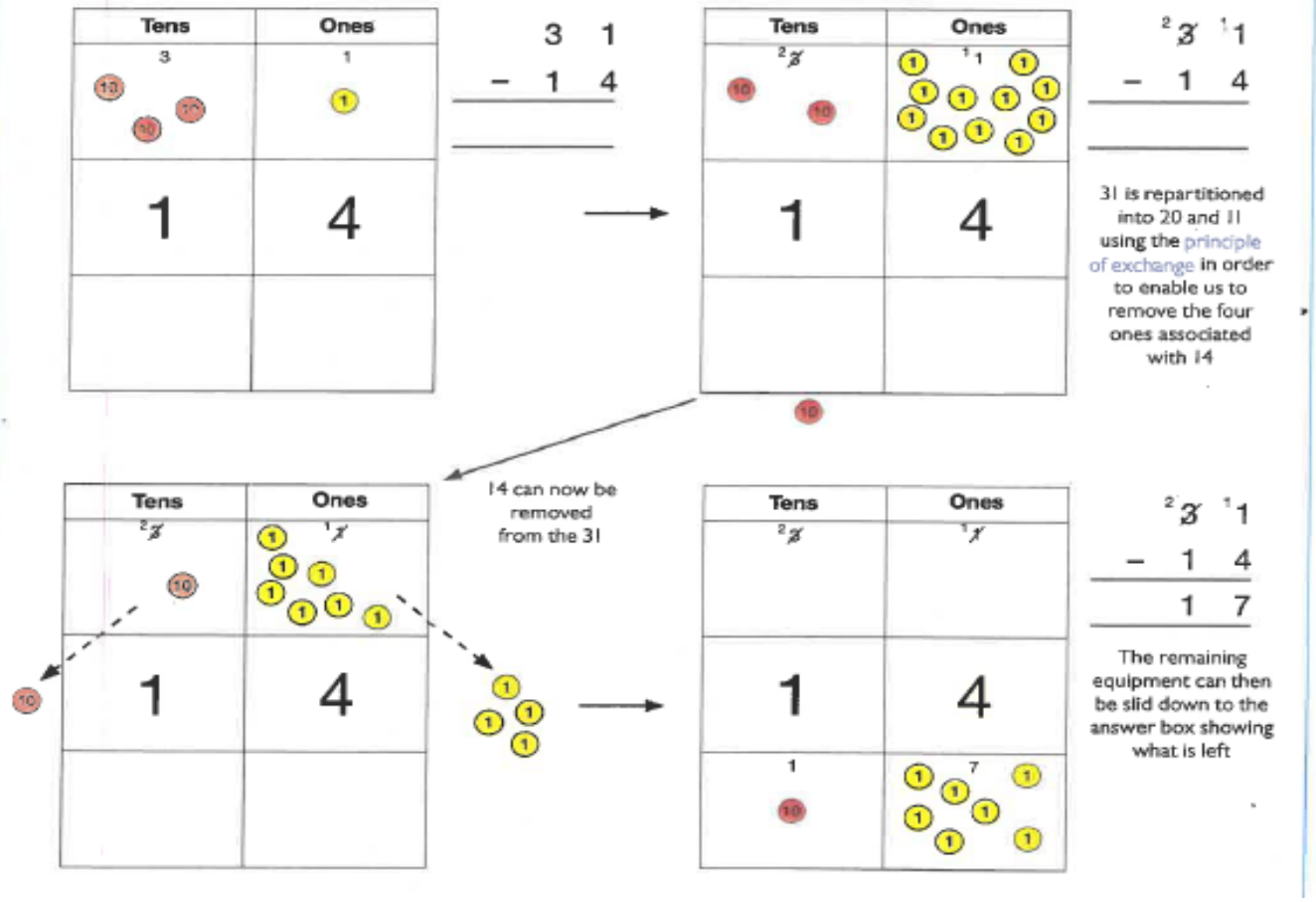
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is../than…?, how much more/less is…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will now be secure in organising their concrete equipment in a vertical manner for subtraction using the principle of exchange appropriately.

They will be now able to make the links between this representation and the formal column subtraction when seen alongside each other.

31 – 14



Stage 6 Addition and Subtraction

Addition stage 6

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

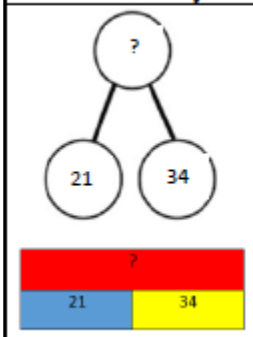
Children will have a full understanding of the links between the concrete representation for column addition and the formal written method.

They will now be able to explore calculating with larger numbers using their understanding of the formal written method.

$$\begin{array}{r} 3\ 2\ 7 \\ +\ 4\ 9\ 6 \\ \hline 8\ 2\ 3 \\ \hline 1\ 1 \end{array}$$

Here are a variety of representations that may be used for addition

Fluency variation, different ways to ask children to solve 21+34:



Sam saved £21 one week and £34 another. How much did he save in total?

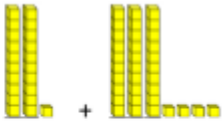
21+34=55. Prove it! (reasoning but the children need to be fluent in representing this)

21
+34
——

21 + 34 =

□ = 21 + 34

What's the sum of twenty one and thirty four?



Always use missing digit problems too:

Tens	Ones
●●	●
●●●	?
?	4

Subtraction stage 6

VOCABULARY **Ensure the correct vocabulary is used at all stages of learning**

subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is../than…?, how much more/less is…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

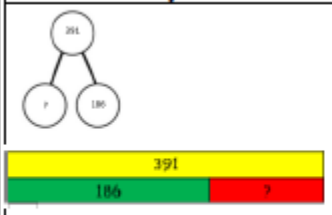
Children will have a full understanding of the links between the concrete representation for column subtraction and the formal written method.

They will now be able to explore calculating with larger numbers using their understanding of the formal written method.

$$\begin{array}{r} 7\ {}^{7}\cancel{8}\ {}^{1}4 \\ -\ 2\ 5\ 9 \\ \hline 5\ 2\ 5 \\ \hline \end{array}$$

Here are a variety of representations that may be used for subtraction

Fluency variation, different ways to ask children to solve 391-186:



Raj spent £391, Timmy spent £186. How much more did Raj spend?

I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?

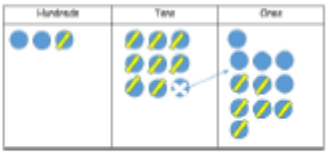
391 - 186

□ = 391 - 186

391
-186
———

Find the difference ebtween 391 and 186
Subtract 186 from 391.
What is 186 less than 391?

What's the calculation? What's the answer?



Stage 1 Multiplication and Division

Multiplication stage 1

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

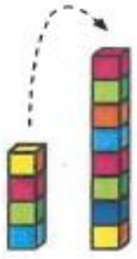
counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide… and so on), repeated addition, array, row, column, double, group in pairs, threes… tens, equal groups of, multiplication, product, inverse

Children will experience practical opportunities involving equal sets or groups using a wide variety of equipment. Practical resources will support children's development of mental pictures and images.

Children will begin to orally count in different multiples including twos, fives and tens making links to natural groupings (e.g. pairs of socks, legs on animals) and the practical resources used.

Children can begin to recognise and continue patterns of multiples using a range of practical resources, e.g. threading beads with two of each colour.

They will begin to use the language and associated representations of doubling.



Double 4 is 8

Division stage 1

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____, ____ equal groups

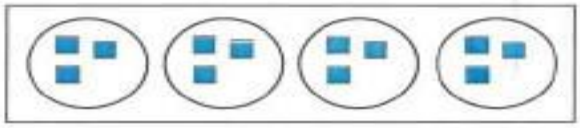
Children will explore the language of sharing. Children will experience practical activities in 'sharing' objects between a small number of groups/people with the emphasis on sharing equally.

Alongside this, with equal weighting, children should be introduced to 'grouping' objects as a representation of division (e.g. 'each person gets 2') with the emphasis on equal groups.

They will begin to use the language and associated representations of halving.

Children can be encouraged to develop ways of recording their findings using pictures.

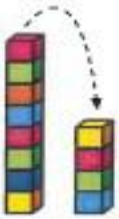
12 shared into 3 equal groups.
12 shared equally into groups of 4.



12 shared into 4 equal groups.
12 shared equally into groups of 3.

6 football stickers shared between 2 people

6 football stickers, how many people can have 2 each?



Half 8 is 4

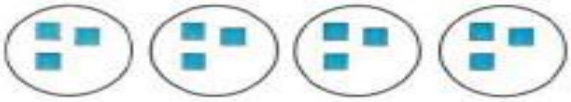
Stage 2 Multiplication and Division

Multiplication stage 2

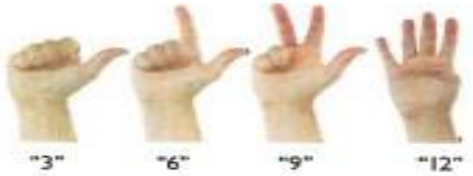
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide… and so on), repeated addition, array, row, column, double, group in pairs, threes… tens, equal groups of, multiplication, product, inverse

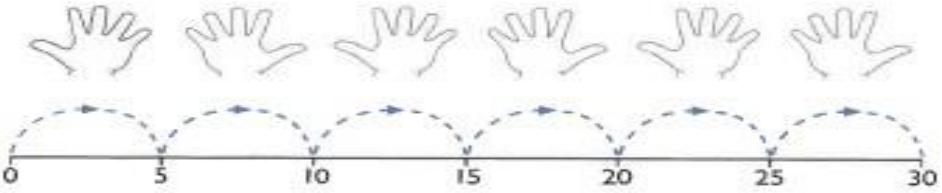
Children will begin to arrange objects into equal groups to aid counting.

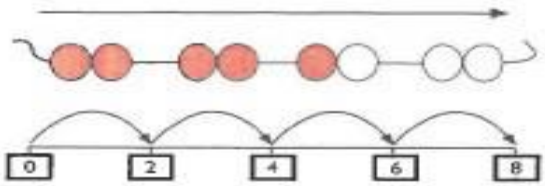


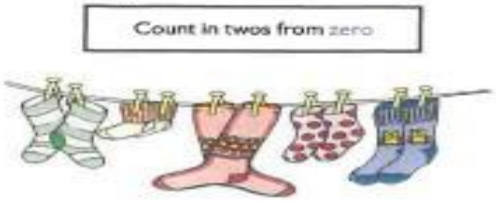
They will continue to count in multiples and begin to relate this to multiplication through finger counting.



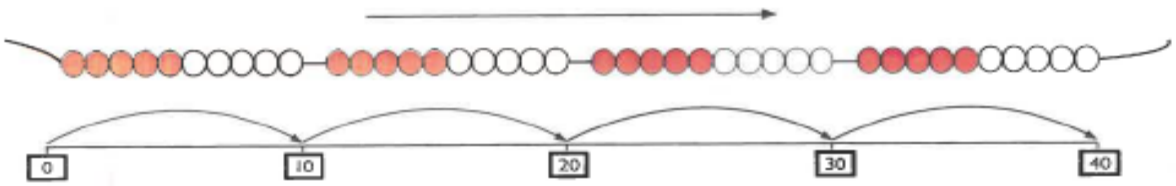
Children will be introduced to a variety of representations of repeated addition; they will see the representations alongside each other and begin to make connections between them.



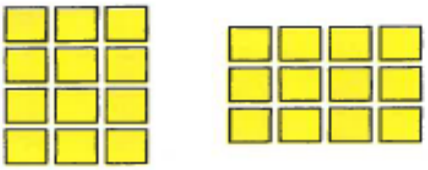




Count in tens from zero



Children will be introduced to the array, using concrete equipment, for small numbers as a way of organising groups to show repeated addition and commutativity. They should explore arrays in the world around us, e.g. egg boxes, baking trays, wrapping papers; and use them to answer questions such as 'How many eggs would we need to fill the egg box?' 'How do you know?'

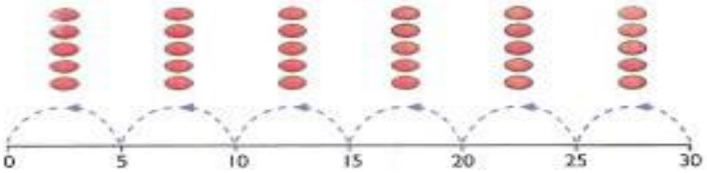


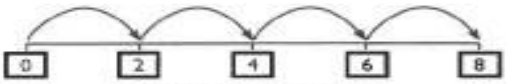
Division stage 2

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

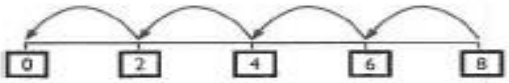
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____, ____ equal groups

Children will relate the grouping of objects to repeated subtraction and begin to represent this using a number line whilst continuing to use concrete equipment.



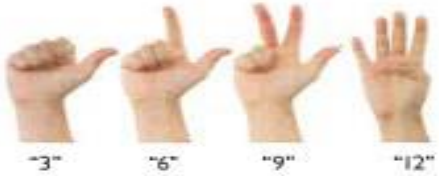


How many 2s are in 8?

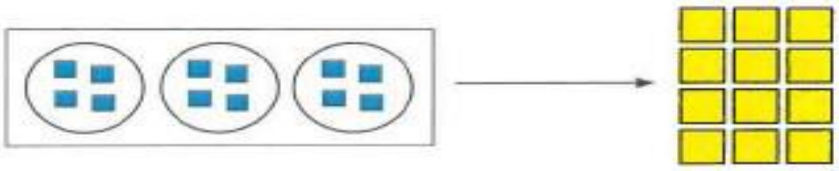


How many 2s can we take away from 8?

Children will use their knowledge of counting up in multiples to solve division calculations and recognise that this in the inverse of multiplication.



Children will continue to group and share equally using concrete equipment and will now begin to organise their groups into an array rather than scattered groupings.



The direct link between multiplication and division should be made explicit when using models and representations.

Children will continue to make links between division and fractions. They will be aware that the division sign is the equivalent to the fraction line and so p÷q can be written as $\frac{p}{q}$.

$1 \div 2$

Stage 3 Multiplication and Division

Multiplication stage 3

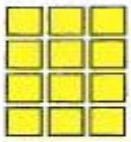
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide… and so on), repeated addition, array, row, column, double, group in pairs, threes… tens, equal groups of, multiplication, product, inverse

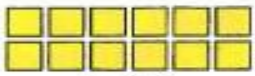
Children will continue to count in multiples and relate this to multiplication through finger counting.

They will be able to model a calculation using a practical array which demonstrates an effective method of counting and the link to repeated addition. Children need to explore related multiplication facts of a given number by making a variety of arrays and explaining what they show.

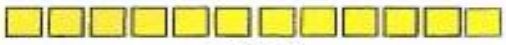
Representing 12



3 × 4 = 12
4 × 3 = 12



2 × 6 = 12
6 × 2 = 12

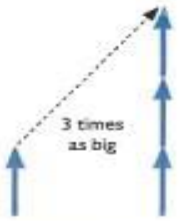


1 × 12 = 12
12 × 1 = 12

3 + 3 + 3 + 3 + 3 = 15

5 + 5 + 5 = 15

The children should be confident with their use of the language of scaling when talking about multiplication.



Division stage 3

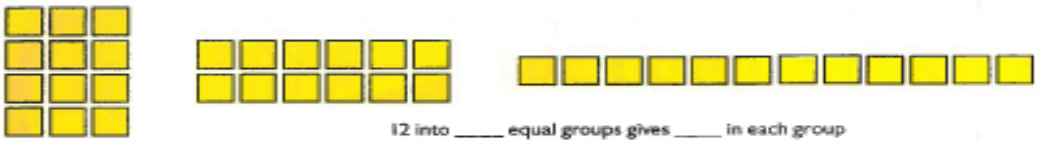
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____ , ____ equal groups

Children will continue to use their knowledge of counting in multiples to support the inverse of multiplication and repeated subtraction.

Children will build on their use of concrete arrays for division recognising the links to repeated subtraction and the inverse of multiplication in order to derive the associated division facts. Children need to explore related division facts of a given number by making a variety of arrays and explaining what they show.

Representing 12



12 into ____ equal groups gives ____ in each group

12 into equal groups of ____ gives ____ groups

The children should be confident with their use of the language of scaling when talking about division with links made to simple fractions (e.g. half the size, three times smaller).

Stage 4 Multiplication and Division

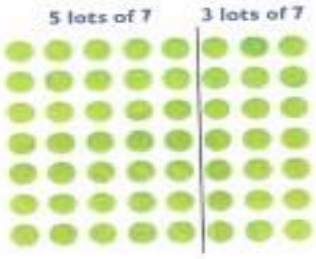
Multiplication stage 4

VOCABULARY **Ensure the correct vocabulary is used at all stages of learning**

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide… and so on), repeated addition, array, row, column, double, group in pairs, threes… tens, equal groups of, multiplication, product, inverse

Children will explore practical arrays for larger numbers. They will think flexibly when working with arrays and will be encouraged to look at arrays beyond repeated addition. They will look for 'friendly' numbers to help them efficiently calculate totals within arrays. E.g. for 7x8… Children may find counting in 7s or 8s tricky but they can look for 'friendly' numbers which are easier to calculate e.g. 4x5, 4x2, 4x5, 4x2.

Thinking flexibly about 7 x 8



Children should continue to experience the language of scaling (e.g. scaling up pictures by multiplying by powers of 10, multiplying by powers of 1000 in converting between units of measure)

Division stage 4

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____ , ____ equal groups

Children will continue to organise groups into an array now working with larger numbers by either grouping or sharing. Children will be able to explain all the facts they know about a given array with no remainder. They should be making arrays with the equipment to establish 'How many in each group?' or 'How many groups?'. Children should continue to experience the language of scaling (e.g. scaling down pictures by dividing by powers of 10, dividing by powers of 1000 in converting between units of measure)

120 ÷ 3

120 shared equally between 3 is 40.
120 shared equally between 4 is 30.
3 equal groups of 40 make 120.
4 equal groups of 30 make 120.

1200 ÷ 3

1200 shared equally between 3 is 400.
1200 shared equally between 4 is 300.
3 equal groups of 400 make 120.
4 equal groups of 300 make 1200.

Stage 5 Multiplication and Division

Multiplication stage 5

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide… and so on), repeated addition, array, row, column, double, group in pairs, threes… tens, equal groups of, multiplication, product, inverse

Children will continue to work with arrays, exploring larger numbers, leading into the grid method of multiplication. Practical experiences may still be required for some children as they enter this stage. To begin with, children should see the array with the grid lines. When appropriate, children should move to using the grid displaying the numbers only.

Children should begin using grid method for 2- and 3-digit by 1 digit numbers and should be given the chance to relate this to facts they know about arrays where needed.

Throughout this stage, children should be encouraged to estimate an approximate answer in order to check for reasonableness and this should become standard practice.

$(6 \times 10) + (6 \times 4)$
$60 + 24$
84

x	10	4
6	60	24

Division stage 5

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____, ____ equal groups

Children will continue to work with concrete arrays, exploring known multiplication/division facts, with the use of grid lines to begin to make the link to short division where numbers are easily divisible. The children understand that the array within short division can be interpreted for both sharing between or equal groups of where the dots within the array each represent 1.

How many equal groups of 7 can I make?
(grouping is represented in the columns)

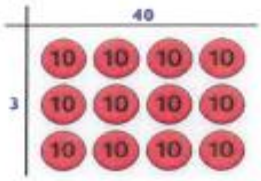
or

If I put these into 7 equal groups, how many in each group?
(sharing between is represented in the rows)

Children will begin to use counters within an array to show the sharing model of division, using their knowledge of the principle of exchange where necessary. At this stage, children are encouraged to consider the links between the sharing model and fractions.

120 can be exchanged for 12 tens in order to make an array

120 shared into 3 equal groups gives 40 in each group



We can explicitly see 40 three times; 3 rows of 40, a ⅓ of 120 is 40.
We can divide the array into three parts and there is 40 in each part.

Stage 6 Multiplication and Division

Multiplication stage 6

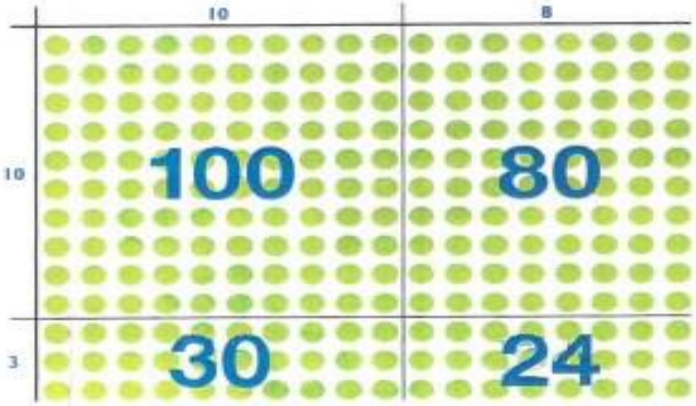
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide… and so on), repeated addition, array, row, column, double, group in pairs, threes… tens, equal groups of, multiplication, product, inverse

Children will now be secure in using the grid method for multiplying by one-digit numbers and will begin to explore the links between the grid method and the expanded method of short multiplication.

$$\begin{array}{rr} & 14 \\ \times & 6 \\ \hline & 24 \quad (6 \times 4) \\ + & 60 \quad (6 \times 10) \\ \hline & 84 \end{array}$$

Children will also begin to explore the use of arrays and the grid method for multiplying by two-digit numbers.



x	10	8
10	100	80
3	30	24

$$\begin{array}{rrr} & 1 & 8 & 0 \\ + & & 5 & 4 \\ \hline & 2 & 3 & 4 \\ & 1 & & \end{array}$$

Division stage 6

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

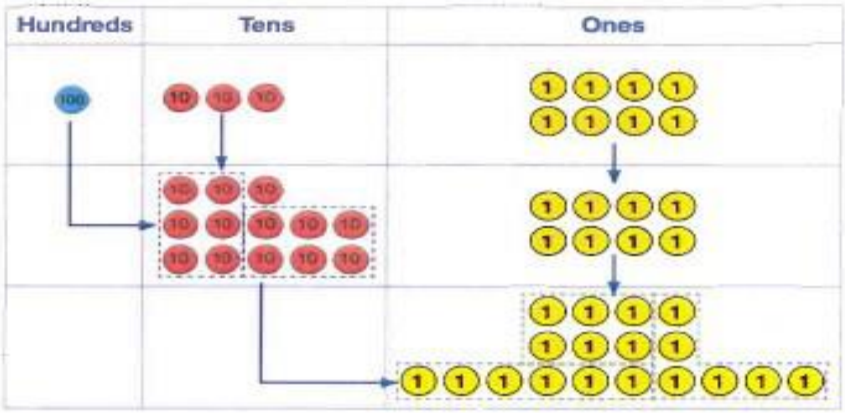
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____, ____ equal groups

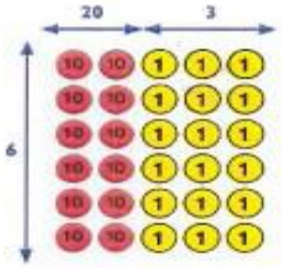
Children will work with equipment to divide any integer by a single digit divisor using their sound knowledge of the principle of exchange.

They will begin to be introduced to numbers that have remainders and will recognise and be able to talk about these when they do not 'fit' into the array.

Children will be introduced to the notation of short division, linking with the principle of exchange and how this relates to the practical representations.

Children continue to be encouraged to consider the links between the sharing model and fractions.





In the array, we can explicitly see 23 six times; 6 rows of 23. This is the sharing model.

⅙ of 138 is 23.

We can divide the array up into six equal parts and there is 23 in each part.

$$6\overline{)\,1^{1}3\;\;1^{1}8}\qquad 23$$

Stage 7 Multiplication and Division

Multiplication stage 7

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide… and so on), repeated addition, array, row, column, double, group in pairs, threes… tens, equal groups of, multiplication, product, inverse

Children will now have a good understanding of the expanded short multiplication method and will begin to represent this as compact short multiplication for TU x U.

```
  1 4
x   6
-----
  2 4   (6 x 4)
+ 6 0   (6 x 10)
-----
  8 4
```

→

```
  1 4
x   6
-----
  8 4
  2
```

Children will be secure in using the grid method for multiplying by two-digit numbers and will begin to explore the links between the grid method and the expanded method of long multiplication.

x	10	8
10	100	80
3	30	24

→

```
    1 8
  x 1 3
-------
    2 4   (3 x 8)
    3 0   (3 x 10)
    8 0   (10 x 8)
+ 1 0 0   (10 x 10)
-------
  2 3 4
  1
```

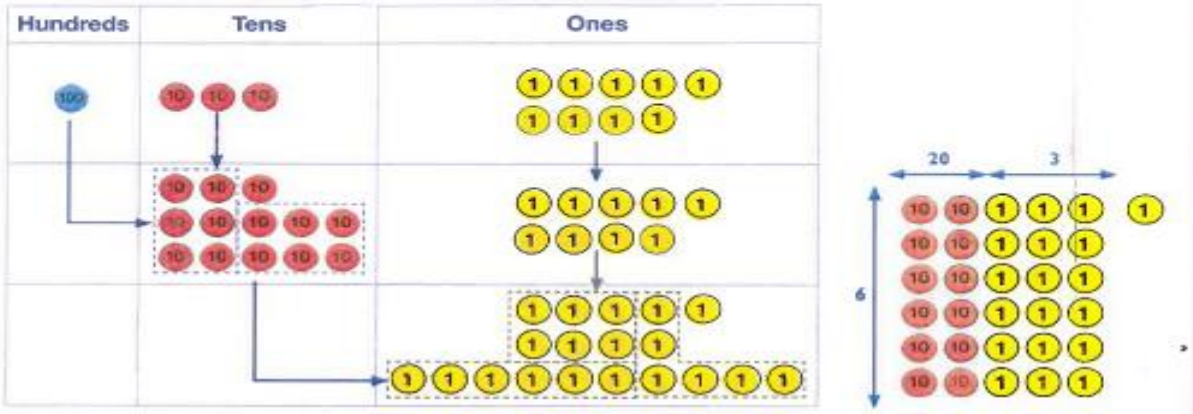
Division stage 7

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____, ____ equal groups

Children will now be secure in using short division for one-digit divisors with an integer quotient.

They will now begin to use the short division notation for calculations involving remainders.



$$\begin{array}{r} 2\ 3\ \text{r}1 \\ 6\overline{)\ 1\!\not{4}\ {}^{1}3\ {}^{1}9} \end{array}$$

Children will also begin to explore the use of jottings of friendly numbers to support long division of calculations with 2-digit divisors.

1 x 15 = 15
2 x 15 = 30
4 x 15 = 60
8 x 15 = 120
10 x 15 = 150

20 x 15 = 300

$$\begin{array}{rrrl} & & 2 & 8 \\ 15\overline{)} & 4 & 2 & 0 \\ - & 3 & 0 & 0 \quad (20 \times 15) \\ \hline & 1 & 2 & 0 \\ - & 1 & 2 & 0 \quad (8 \times 15) \\ \hline & & 0 & \end{array}$$

Stage 8 Multiplication and Division

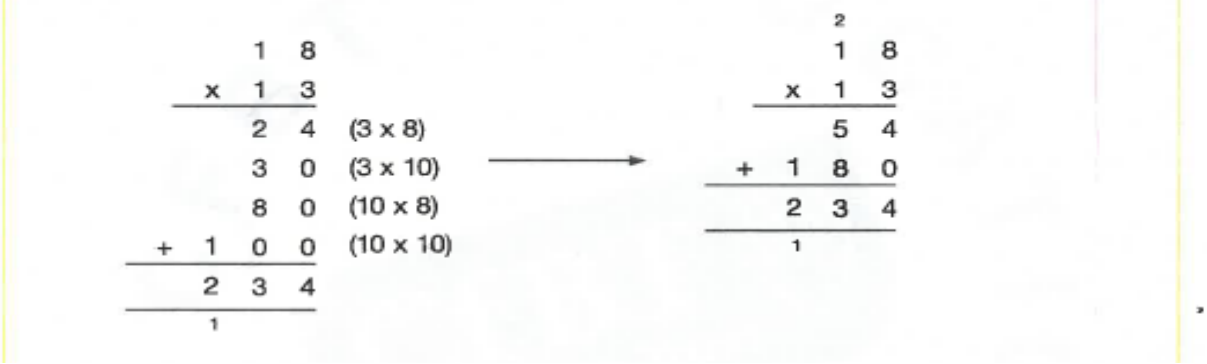
Multiplication stage 8

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide… and so on), repeated addition, array, row, column, double, group in pairs, threes… tens, equal groups of, multiplication, product, inverse

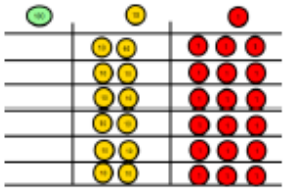
Children will now have a good understanding of the short multiplication method.

Children will now have a good understanding of the expanded long multiplication method and will begin to represent this as compact long multiplication.



Here are a variety of representations that may be used for multiplication

Fluency variation, different ways to ask children to solve 6 x 23:

23 23 23 23 23 23 ? With the counters, prove that 6 x 23 = 138 Why is 6 x 23 = 32 x 6?	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? Tom saved 23p three days a week. How much did he save in 2 weeks?	Find the product of 6 and 23 6 x 23 = □ = 6 x 23 6 × 23 23 × 6	What's the calculation? What's the answer? 

Division stage 8

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____, ____ equal groups

Children will now be secure in using short division for one-digit divisors and long division for two-digit divisors with an integer quotient.

They will now explore the use of long division for two-digit divisors which may include a remainder.

The children will begin to interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context.

1 x 15 = 15
2 x 15 = 30
4 x 15 = 60
8 x 15 = 120
10 x 15 = 150

20 x 15 = 300

```
      2  8 r12
15 | 4  3  2
   - 3  0  0   (20 x 15)
     1  3  2
   - 1  2  0   (8 x 15)
        1  2
```

```
      2  8 12/15
15 | 4  3  2
   - 3  0  0   (20 x 15)
     1  3  2
   - 1  2  0   (8 x 15)
        1  2
```

```
      2  8 4/5
15 | 4  3  2
   - 3  0  0   (20 x 15)
     1  3  2
   - 1  2  0   (8 x 15)
        1  2
```

Here are a variety of representations that may be used for division

Fluency variation, different ways to ask children to solve 615 ÷ 5:

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?  	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	$5\overline{)615}$ 615 ÷ 5 = ☐ = 615 ÷ 5 How many 5's go into 615?	What's the calculation? What's the answer?